OCR

RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level

former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS 5509

Mechanics 3

Thursday **21 JUNE 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
Students' Handbook

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/ answer booklet.

Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown on the answer paper to indicate that a correct method is being used.

Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.

This question paper consists of 4 printed pages.

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[Turn over

1 A child suspends a small model aeroplane from the ceiling by means of a light elastic string of modulus 5 N and natural length 0.8 m. The aeroplane is packed with some modelling clay and the total mass is 0.1 kg.

(i) Calculate the length of the string when the aeroplane hangs in equilibrium. [3]

(ii) The aeroplane is pulled vertically down until the string is 1.2 m long and then released from rest. Calculate the minimum distance between the aeroplane and the ceiling in the resulting motion. [4]

The child wishes the aeroplane to hang 0.9 m below the ceiling when in equilibrium. The child considers two possibilities. The first possibility is to remove some of the clay.

(iii) What mass of clay would need to be removed? [2]

The other possibility is not to remove any clay, but instead to shorten the string to a new natural length.

(iv) What would the new natural length need to be? [3]

The child again holds the aeroplane 1.2 m below the ceiling and releases it from rest.

(v) Explain, without doing any calculations, whether the aeroplane will get closer to the ceiling than in part **(ii)**,

(*A*) if clay has been removed as in part **(iii)**,

(*B*) if the string has been shortened as in part **(iv)**. [3]

[Total: 15]

2 A piston performs simple harmonic motion with amplitude 0.1 m about a point O. The displacement from O is denoted by x metres and the time by t seconds. In a test, the piston is first observed at $t = 0$ when $x = 0.05$ and it is moving *towards* O.

(i) Sketch a graph of x against t for one complete oscillation starting at $t = 0$. [3]

An expression for x of the form $x = a\sin(\omega t + \varepsilon)$ is sought, where a, ω and ε are positive constants.

(ii) Write down the value of a and find the value of ε. [4]

The piston passes through O after 0.025 seconds.

(iii) Calculate ω and the period of the motion. [4]

(iv) Calculate the acceleration of the piston when it is first observed. [3]

[Total: 14]

3 A governor is a device to limit the angular velocity of an engine. A simple design consists of a light rod, AB, of length 0.1 m, with a small brass sphere of mass 0.2 kg attached at B. The end A is freely hinged to a vertical axle.

The sphere moves in a horizontal circle with angular velocity ω rad s^{-1} and the angle that AB makes with the downward vertical is θ, as shown in Fig. 3.1.

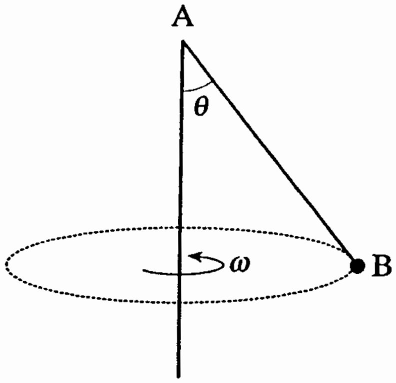


Fig. 3.1

(i) Write down the vertical equilibrium equation and the radial equation of motion for the sphere. [5]

The governor is designed to act when the angular velocity reaches 20 rad s^{-1}.

(ii) Calculate the value of θ when $\omega = 20$. [2]

The design is modified by the addition of a light rod BC of length 0.1 m freely hinged to the first rod at B. The other end is attached to a smooth ring C of mass 0.1 kg which can slide on the axle as shown in Fig. 3.2. Both rods are inclined at the same angle, ϕ, to the vertical.

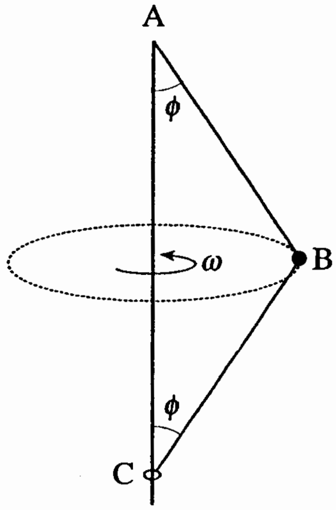


Fig. 3.2

(iii) Calculate the value of ϕ when the angular velocity is 20 rad s^{-1}. [8]

[Total: 15]

[Turn over

4 **(a)** A region in the first quadrant is bounded by the curve $x^2 + 4y^2 = 4$ and the axes. A uniform solid is obtained by rotating this region through 2π radians about the x-axis, as shown in Fig. 4.

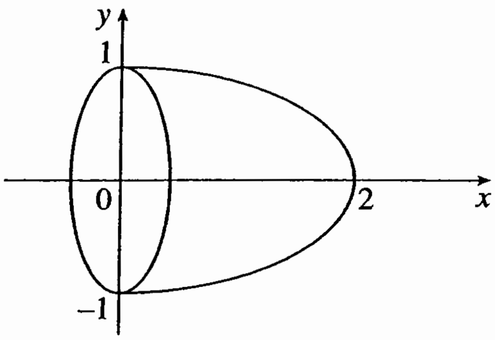


Fig. 4

Calculate, by integration, the x-coordinate of the centre of mass of the solid. [6]

(b) Bullets of various sizes are made in the shape of the solid in part **(a)**. An attempt is made to model the force of air resistance against the bullet as it flies through the air. It is suggested that the resistance, R, is proportional to a product of powers of the radius of the flat end of the bullet, r, the velocity of the bullet, v, the viscosity of the air, η, and a dimensionless constant.

(i) Given that the dimensions of viscosity are $\mathrm{ML^{-1}T^{-1}}$, use dimensional analysis to find an expression for R. [6]

It is suggested that the density of the air should also have been included.

(ii) Explain briefly why it would not be possible to determine the exact form of R in terms of radius, velocity, viscosity and density by dimensional analysis. [2]

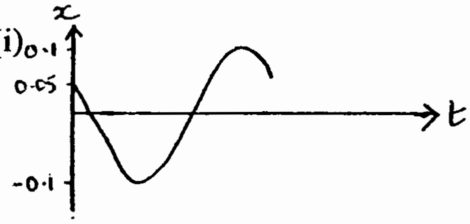
(iii) A suggested model is $R = k r^{\alpha} v^{\beta} \eta^{\gamma} \left(\frac{\rho r v}{\eta} \right)^{\delta}$, where ρ is the density of the air and k is a dimensionless constant. Show that the value of δ does not affect the dimensional consistency of this model. [2]

[Total: 16]

Mark Scheme

1.(i)	$0.1g = \frac{5x}{0.8}$	M1	use of Hookes law.
	$x = 0.1568$	M1	solving
	length $= 0.9568 \approx 0.96$m	A1	
(ii)	$\frac{5\times0.4^2}{2\times0.8} = 0.1gh$	M1	energy equation including EPE
		M1	reasonable attempt at all terms
		A1	correct equation
	$h = 0.51$ m		
	min. distance $= 1.2 - 0.51 = 0.69$ m	A1	
(iii)	$mg = \frac{5\times0.1}{0.8} \Rightarrow m = 0.0638$	M1	
	so remove 0.036 kg	A1	
(iv)	$\frac{5(0.9-l_1)}{l_1} = 0.1g$	M1	equation for natural length or extension
		A1	
	solving $\Rightarrow l_1 = 0.75$ m	A1	
(v)	$mgh = \frac{\lambda x^2}{2l_0}$		
	(A) $m \downarrow \Rightarrow h \uparrow$ i.e. closer	B1	
	(B) $x \uparrow, l_0 \downarrow \Rightarrow$ EPE $\uparrow$	M1	
	$\Rightarrow h \uparrow$ i.e. closer	A1	convincing argument

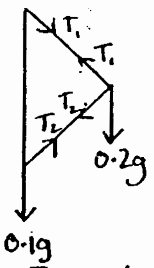
2.(i)



2.(i)	(graph)	B1	shape (one oscillation only)
		B1	max/min at ±0.1
		B1	$x = 0.05$ and negative gradient at $t = 0$
(ii)	$a = 0.1$	B1	
	$0.1\sin\varepsilon = 0.05$	M1	use of $t = 0$, $x = 0.05$ and their a
		M1	solving
	negative gradient $\Rightarrow \varepsilon = \frac{5\pi}{6}$	A1	cao
(iii)	$0.1\sin(0.025\omega + \frac{5\pi}{6}) = 0$	M1	$x = 0$, $t = 0.025$ and their a, ε
	$0.025\omega + \frac{5\pi}{6} = \pi$	M1	solving
	$\omega = \frac{20\pi}{3}$	A1	
	$T = \frac{2\pi}{\omega} = 0.3$ s	F1	
(iv)	$\ddot{x} = -\omega^2 x$ $\left[\text{or } \ddot{x} = -a\omega^2\sin(\omega t + \varepsilon)\right.$		
	$= -\left(\frac{20\pi}{3}\right)^2 0.05$ $\left. = -0.1\left(\frac{20\pi}{3}\right)^2 \sin\frac{5\pi}{6}\right]$	M1	substitute values
	$= -21.9$ m s^{-1}	A1	negative acceleration
		A1	21.9 cao

3.(i)	$T\cos\theta = 0.2g$	B1	
	$T\sin\theta = 0.2r\omega^2$	M1	N2L with $r\omega^2$ or v^2/r
		A1	
	$r = 0.1\sin\theta$	M1	eliminate r
	$\Rightarrow T = 0.02\omega^2$	A1	(these marks may be scored in (ii))
(ii)	$\omega = 20 \Rightarrow T = 8 \Rightarrow \cos\theta = \frac{0.2g}{8}$	M1	eliminate T
	$\theta = 75.8°$	A1	

(iii)



(iii)	$T_1\cos\phi - T_2\cos\phi = 0.2g$	B1	
	$T_1\sin\phi + T_2\sin\phi = 0.2r\omega^2$	M1	good attempt at all terms
		A1	
	$T_2\cos\phi = 0.1g$	B1	
	$T_1\cos\phi = 0.3g$	M1	eliminate T_2
	$T_1 + T_2 = 0.1\times 0.2\times 20^2 = 8$	M1	eliminate r
	$8\cos\phi = 0.4g$	M1	eliminate T_1
	$\phi = 60.7°$	A1	
4.(a)	$V = \int_0^2 \pi(1-\frac{x^2}{4})\mathrm{d}x = \frac{4\pi}{3}$	M1	attempt volume
		A1	
	$\frac{4\pi}{3}\bar{x} = \int_0^2 \pi(x - \frac{x^3}{4})\mathrm{d}x$	M1	use of formula (including LHS or equivalent)
	$= \pi\left[\frac{x^2}{2} - \frac{x^4}{16}\right]_0^2 = \pi$	M1	integrating
		A1	
	$\bar{x} = \frac{3}{4}$	A1	
(b)(i)	$R = kr^\alpha v^\beta \eta^\gamma$	M1	
	$MLT^{-2} = L^\alpha(LT^{-1})^\beta(ML^{-1}T^{-1})^\gamma$	A1	
	$1 = \gamma$		
	$1 = \alpha + \beta - \gamma$	M1	two equations
	$-2 = -\beta - \gamma$	M1	third equation
	$\alpha = \beta = \gamma = 1$	A1	
	$R = krv\eta$	A1	
(ii)	M, L, T give 3 equations	B1	
	but 4 unknowns so cannot solve	B1	
(iii)	$\left[\frac{\rho r v}{\eta}\right] = \frac{ML^{-3}.L.LT^{-1}}{ML^{-1}T^{-1}} = 1$ i.e. dimensionless	B1	must be shown
	so unaffected by the value of δ	B1	

Examiner’s Report

Mechanics 3 (5509)

General Comments

There were many good responses to this paper. However, the common problems of insufficient working for a given result, of unclear working and a lack of clear diagrams hampered many candidates.

Comments on Individual Questions

Question 1 (Model aeroplane: Hooke's Law)

The first part of this question was often well answered. Although there were many good answers to the second part, there were also many errors here. Many candidates unnecessarily split the motion into two stages, many omitted to include gravitational potential energy and many used the displacement from the equilibrium position as the extension in the elastic potential energy term. Some candidates ignored energy and attempted to use simple harmonic motion, rarely with success.

Few candidates had problems calculating the new mass in the third part, although some omitted to calculate the mass to be removed, and some made errors in this simple subtraction.

Most candidates made errors in calculating the new natural length, usually omitting to express the extension in terms of the length.

The explanations asked for in the final part were often vague or lacking in detail or argued from false assumptions. Simple arguments relating to energy were the most successful here, although misconceptions as to which quantities had changed and which remained the same abounded.

(i) 0.96 m; (ii) 0.69 m; (iii) 0.036 kg; (iv) 0.75 m.

Question 2 (Simple harmonic motion)

The sketch graph was often well drawn, but frequently candidates ignored the instruction to draw just one oscillation, or omitted one of the details given in the question. Most candidates could write down the correct value of a and of $\sin\varepsilon$ but few realised that ε had to be in the second quadrant to be consistent with the initial direction of motion (and the shape of the graph). Most candidates were also unable to calculate ω correctly, usually setting $0.025\omega + \varepsilon$ equal to 0 rather than π. Candidates usually were able to use their value of ω to calculate the period and knew how to calculate the acceleration although many neglected the direction, and so gave a positive answer.

(ii) 0.1, $5\pi/6$; (iii) $20\pi/3$, 0.3 s; (iv) -21.9 m s^{-2}.

Question 3 (Circular motion of a governor)

The first two parts of this question were often done well, although it was surprising to see a significant minority could not produce this standard analysis of a conical pendulum. The third part was sometimes done well but more commonly done badly. Candidates were hampered by the lack of a good diagram, by not considering the vertical equation at C and often by wrongly assuming that the tensions in the rods were equal.

(i) $T = 0.02\omega^2$; (ii) 75.8°; (iii) 60.7°.

Question 4 (Centre of mass and dimensions)

This question was often well done. Algebraic skills let some candidates down when calculating the centre of mass but most were able to do this correctly. For the dimensional analysis, some candidates did not consider raising the quantities to an unknown power and assumed that the resistance was merely proportional to the radius, velocity and viscosity. As this is in fact the correct expression they were able to verify this, but this approach did not answer the question asked and so received only partial credit.

Many candidates did not seem to realise that the introduction of another quantity leads to four unknown indices but only three dimensional equations and therefore no unique solution, but a good number made some attempt along these lines to explain the problem of introducing density. Most candidates showed that the bracketed expression in the last part was dimensionless, but often did not make any conclusion about δ, as asked.

(a) ¾; (b)(i) $R = krv\eta$.